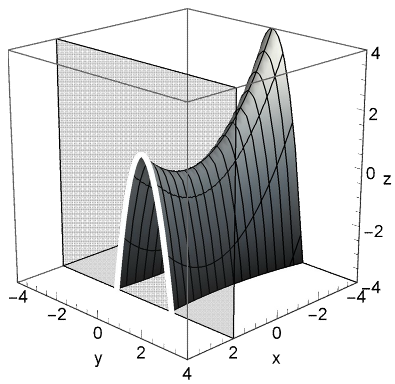
1) The equation of the slice curve on the graph of $z = \frac{x^2}{2} - 3\,y^2$ shown below is:

a) $z = 2\,x^2 - \frac{3}{2}\,y^2$ b) $z = \frac{x^2}{2} - 3$ c) $z = \frac{x^2}{2} - 12$ d) $z = 4 - 3\,y^2$ e) $z = 2 - 3\,y^2$



FORM A

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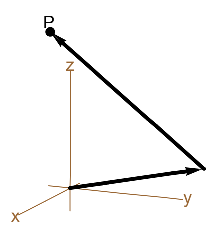
2) The vector $X = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ is shown with the vector $V = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ drawn with its tail at the tip of X. The tip of the displaced vector V points to a point P. What vector Y with its tail at the origin points to P? $Y =$



a) $\frac{V \cdot X}{|V|\,|X|}\,V$ b) $X \times V$ c) $V - X$ d) $V + X$ e) $\frac{V \cdot X}{|V|\,|X|}\,X$

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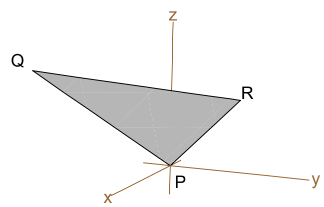
3) The steepest slope of a line on the explicit planar graph of $\boldsymbol{z = 12\,x - 5\,y}$ is?

a) $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$ b) 17 c) 7 d) 13 e) $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

Problems 4, 5, 6 use the triangle with vertices at: $P = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $Q = \sqrt{2} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $R = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$



4) What is the length of the QR side of the triangle PQR?

a) $\sqrt{3}$ b) $2\sqrt{2}$ c) $2\sqrt{3}$ d) $3\sqrt{2}$ e) $4\sqrt{2}$

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5) What is the area of the triangle PQR?

a) $\sqrt{3}$ b) $2\sqrt{2}$ c) $2\sqrt{3}$ d) $3\sqrt{2}$ e) $4\sqrt{2}$

6) What is the angle at vertex R?

a) 30˚ b) 45˚ c) 60˚ d) 90˚ e) 120˚

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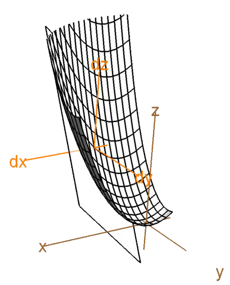
7) The equation of the explicit plane tangent to $z = \frac{x^2}{4} + \frac{y^2}{3}$ at $(x, y, z) = (2, -3, 4)$ is:

a) $(z - 4) = \frac{x}{2}(x - 2) + \frac{2y}{3}(y + 3)$ b) $(z - 4) = \frac{x}{2}(x - 3) + \frac{2y}{3}(y + 2)$

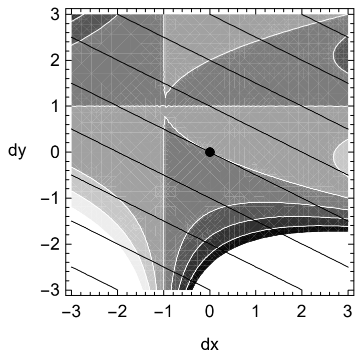
c) $(z - 4) = (x - 2) - 2(y + 3)$

d) $(z - 4) = (x + 2) - 2(y - 3)$ e) $(z - 4) = 2(x - 2) - (y + 3)$



8) The gradient of $f[x, y] = x\,y^3 - x^2\,y$ at the point $(x, y) = (1, -1)$ is:

a) $G\,(1, 2)$ b) $G(1, 3)$ c) $G(-3, 2)$ d) $G(-3, 1)$ e) $G(4, -3)$



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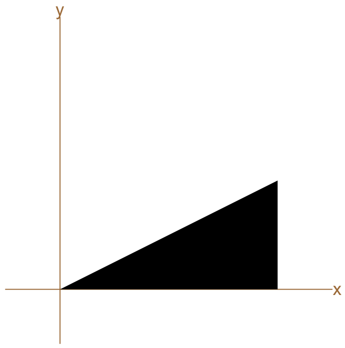
9) Calculate the area-weighted integral $\int_{\mathbb{D}} f\, da$ over the rectangle $\mathbb{D} = \{(x, y) : -1 \le x \le 3,\; 1 \le y \le 2\}$

where $f[x, y] = 3\,x^2\,y$, $\int_{\mathbb{D}} f\, da =$

a) 7 b) 18 c) 28 d) 42 e) 84

10) The area-weighted integral $\int_{\mathbb{D}} x\, y^2\, da$ over the region $\mathbb{D}$ bounded by the curves $y = x/2$, $y = 0$, $x = 2$ shown below is:

a) $\frac{4}{15}$ b) $\frac{2}{5}$ c) $\frac{2}{3}$ d) $\frac{4}{3}$ e) 1



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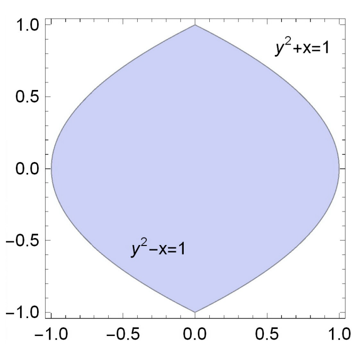
11) Set up the area weighted integral of the function $f[x, y] = 2x^2 y$ over the region bounded by $y^2 + x = 1$ and $y^2 - x = 1$.

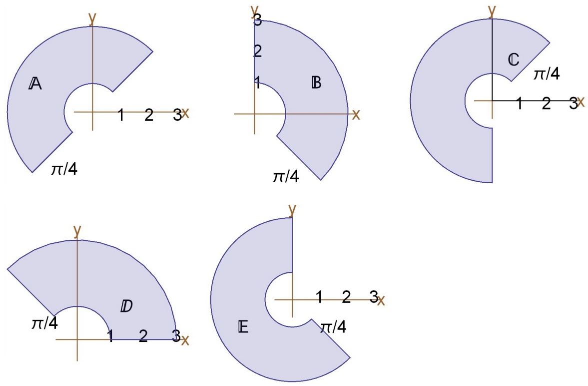
a) $\int_{-1}^{1}\int_{\sqrt{1-x}}^{\sqrt{1+x}} 2x^2 y\, dy\, dx$ b) $\int_{-1-y^2}^{1-y^2}\int_{-1}^{1} 2x^2 y\, dy\, dx$ c) $\int_{-1}^{1}\int_{y^2-1}^{1-y^2} 2x^2 y\, dy\, dx$

d) $\int_{-1}^{1}\int_{y^2-1}^{1-y^2} 2x^2 y\, dx\, dy$ e) $\int_{-1}^{1}\int_{1-y^2}^{y^2-1} 2x^2 y\, dx\, dy$



12) The iterated integral $\int_1^3\left(\int_{\pi/4}^{3\pi/2} f[r\,\mathrm{Cos}[\theta],\, r\,\mathrm{Sin}[\theta]]\, r\, d\theta\right) dr$ equals the 2D area-weighted integral $\int_{\mathbb{D}} f[x,\, y]\, da$ for which of the domains:

a) *A* b) *B* c) C d) *D* e) E



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13) A particle moves with position $X[t]$, velocity $V[t] = X'[t]$, and acceleration $A[t] = V'[t]$.
Such a motion lies on a sphere ($|\,X[t]\,| = r$, constant, for all t) when:

a) acceleration is constant b) velocity is constant c) speed is constant

d) $X[t]$ is perpendicular to $V[t]$ for all t e) $V[t]$ is perpendicular to $A[t]$ for all t

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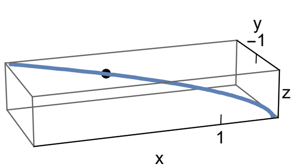
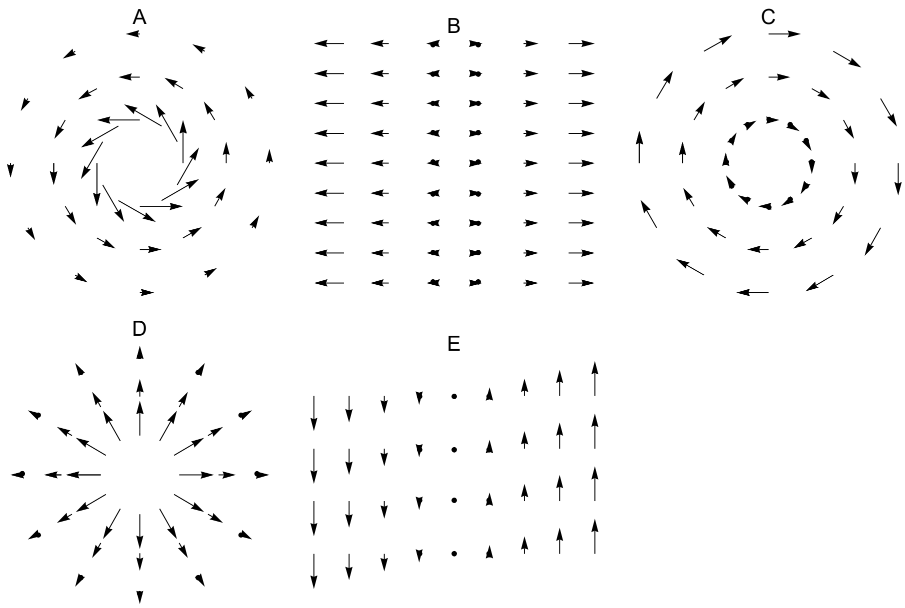
14) A parametric curve is given by $X[t] = \frac{1}{6}\begin{pmatrix} t^4 \\ t^3 \\ t^2 \end{pmatrix}$ A tangent vector at the point where $t = -2$ is:

a) $\frac{1}{3}\begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}$ b) $\frac{1}{3}\begin{pmatrix} -16 \\ 6 \\ -2 \end{pmatrix}$ c) $\frac{1}{3}\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ d) $\begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix}$ e) $\frac{1}{6}\begin{pmatrix} -16 \\ 8 \\ -4 \end{pmatrix}$



15) Which is the vector field for $\boldsymbol{F}[X] = \begin{pmatrix} 0 \\ x \end{pmatrix}$



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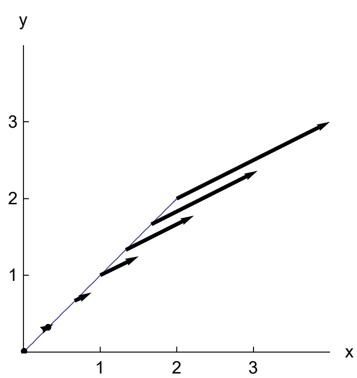
16) The work done (or flow along) by the vector field $\boldsymbol{F}[X] = \begin{pmatrix} 2xy \\ x^2 \end{pmatrix}$ moving along the line $y = x$ from (0, 0) to (2, 2) is:

a) 16   b) 1 c) -1 d) -8 e)8



17) Find a potential function $p[x,\ y]$ for the vector field $\boldsymbol{F}[X] = \begin{pmatrix} 2\,y + 2\,x \\ 2\,x\ + 1 \end{pmatrix}$.

a) $2\,x\,y + x^2 + y$ b) $2\,x\,y$ c) $x^2\,y^2 + y$ d) $2\,x\,y + x^2$ e) $F[X]$ has no potential function.

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18) Which vector field is not given by a potential, that is, has ***no*** function $p[x,\ y]$ with $\nabla p[x,\ y] = F[x,\ y]$

a) $F[x,\ y] = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$ b) $F[x,\ y] = \begin{pmatrix} -y \\ x \end{pmatrix}$ c) $F[x,\ y] = \begin{pmatrix} y \\ x \end{pmatrix}$ d) $F[x,\ y] = \begin{pmatrix} x \\ 0 \end{pmatrix}$ e) $F[x,\ y] = \begin{pmatrix} 0 \\ y \end{pmatrix}$

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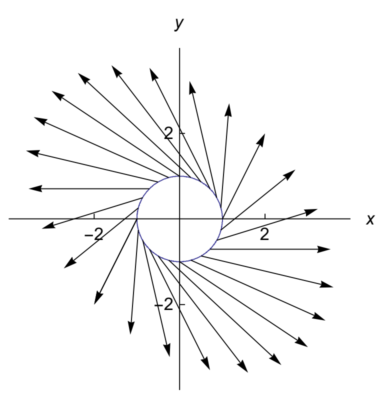
19) For the field $F[X] = \begin{pmatrix} x - 3\,y \\ 2\,x + 2\,y \end{pmatrix}$. The flows for the unit circle are as follows:

a) Out = 5π, Around =3π b) Out = 3π, Around =5π c) Out = 4π, Around = -2π

d) Out = -2π, Around = 4π e) Out =3π, Around = -π



20) Consider the series $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \ldots$ Does this series converge? And if so, evaluate the series.

a) $\frac{5}{3}$ b) $\frac{3}{5}$ c) $\frac{3}{2}$ d) $\frac{2}{3}$ e) This series does not converge.

$$\frac{2}{5}\left(1 + \frac{2}{5} + \frac{4}{25} + \cdots\right) = \frac{2}{5}\sum_{k=0}^{\infty}\left(\frac{2}{5}\right)^k$$

$\frac{2}{5} < 1 \Rightarrow$ converges

Geometric Series $\Rightarrow$ $\frac{2}{5}\left(\frac{1}{1-2/5}\right) = \frac{2}{5}\left(\frac{1}{3/5}\right) = \frac{2}{3}$

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21) Find the power series for $F[x] = \frac{1}{3-2x}$.

a) $\sum_{k=0}^{\infty} \left(\frac{2}{9}\right)^k x^k$ b) $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right) x^k$ c) $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^k x^k$

d) $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^k x^k$ e) $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k x^k$

Hint: $\frac{1}{3-2x} = \frac{1}{3}\left(\frac{1}{1-\frac{2}{3}x}\right)$ - OR - use Taylor's formula

$$\frac{1}{3-2x} = \frac{1}{3} \cdot \frac{1}{1-\frac{2}{3}x} = \frac{1}{3}\sum_{k=0}^{\infty}\left(\frac{2}{3}x\right)^k$$

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22) If $i = \sqrt{-1}$, the complex number, the exact numerical value of $e^{i\pi} + \sum_{k=0}^{\infty} \frac{1}{2^k}$ =

a) 0 b) $\frac{1}{2}$ c) 1 d) $i + 1$ e) 2

Careful!

$$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1 + 0 = -1$$

$$\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{1-1/2} = \frac{1}{1/2} = 2$$

$$-1 + 2 = 1$$

23) Find the radius of convergence of the series $\text{Log}\left[1+\frac{x}{3}\right]=\frac{x}{3}-\frac{x^2}{18}+\frac{x^3}{81}-\frac{x^4}{324}+\ldots=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}\left(\frac{x}{3}\right)^k$

a) 0 b) 1 c) ∞ d) 3 e) $\frac{1}{3}$

$$r=\lim_{k\to\infty}\left|\frac{\frac{(-1)^{k-1}}{k}\cdot\left(\frac{1}{3}\right)^k}{\frac{(-1)^k}{k+1}\cdot\left(\frac{1}{3}\right)^{k+1}}\right|=\lim_{k\to\infty}\frac{1}{(1/3)}\left|\frac{\frac{1}{k}}{\frac{1}{k+1}}\right|$$

$$=\lim_{k\to\infty}3\left|\frac{k+1}{k}\right|$$

$$=3$$

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24) Differentiate the series $\frac{1}{1+x^2}=\sum_{k=0}^{\infty}(-1)^k x^{2k}$ term-by-term to find the series for $-\frac{2x}{(1+x^2)^2}=$

a) $\sum_{k=0}^{\infty}(-1)^{k-1}k\,x^{2k-1}$ b) $\sum_{k=0}^{\infty}(-1)^k 2k\,x^{2k-1}$ c) $\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{2k+1}$

d) $\sum_{k=0}^{\infty}(-1)^k 2k\,x^{2(k-1)}$ e) $\sum_{k=0}^{\infty}(-1)^k 2k\,x^k$

$$\frac{d}{dx}\left((-1)^k x^{2k}\right)=(-1)^k\cdot 2k\cdot x^{2k-1}$$

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25) Given that $f[x]=\sum_{k=0}^{\infty}(-1)^k\frac{x^k}{(k+1)}$. Find $f'''[0]$.

a) $-\frac{1}{4}$ b) $-\frac{3}{4}$ c) $-\frac{3}{2}$ d) $\frac{1}{4}$ e) $\frac{3}{2}$

$$f(x)=1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+\cdots$$

$$f'''(0)=-\frac{3!}{4}$$

$$=-\frac{6}{4}=-\frac{3}{2}$$