Final Exam Study Guide

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Below is a list of recent material expected on the final exam. Lecture notes can be found under *Files* in the lecture's ICON module.

Final exam notes:

1 Final exam date: Monday 12/13/2021. The final will become available on ICON at 7:00 AM and close at 8:00 PM. See https://registrar.uiowa.edu/final-exam-schedules/fall-2021 for your other final exams.

2 The final exam is worth 33.5% of your final grade. Pop quizzes from lecture will be worth 1.5% of your final grade.

3 The final exam is cumulative, with an emphasis on material covered after midterm 2. Expect roughly 30 questions.

4 Basic calculators and scientific calculators are allowed, but not graphing calculators.

5 If you have not already alerted me to an SDS accommodation, please let me know as soon as possible.

6 This exam requires LockDown Browser. Click here for setup support.

Warning: The below study guide does not include all material that is in our final exam. Please check ICON for any changes to this file.

Quiz 10 material

(1) Section 6.3 - Implicit Differentiation

  (a) explicit function
  (b) implicit function
  (c) implicit differentiation
  (d) tangent line

(2) Section 6.5 - Differentials: Linear Approximation

  (i) differential of x, written dx
  (ii) differential of y, written dy, is the product of $f'(x)dx$
  (iii) linear approximation
  (iv) marginal analysis
  (v) error estimation

(3) Section 7.1 - Antiderivatives

  (i) antidifferentiation and finding the antiderivative
  (ii) indefinite integral
  (iii) power rule for indefinite integrals
  (iv) linearity of indefinite integrals
  (v) indefinite integrals of exponential functions
  (vi) indefinite integral of x^{-1}
  (vii) basic trigonometric integrals

Quiz 11 material

(4) Section 7.2 - Substitution

  (i) differentiating using the chain rule
  (ii) integration by substitution

Substitution

Each of the following forms can be integrated using the substitution $u = f(x)$.

Form of the Integral	Result
1. $\int [f(x)]^n f'(x)\,dx, \quad n \neq -1$	$\int u^n\,du = \frac{u^{n+1}}{n+1} + C = \frac{[f(x)]^{n+1}}{n+1} + C$
2. $\int \frac{f'(x)}{f(x)}\,dx$	$\int \frac{1}{u}\,du = \ln\lvert u\rvert + C = \ln\lvert f(x)\rvert + C$
3. $\int e^{f(x)} f'(x)\,dx$	$\int e^u\,du = e^u + C = e^{f(x)} + C$

Substitution Method

In general, for the types of problems in this book, there are four cases. We choose u to be one of the following:

1. the quantity under a root or raised to a power;
2. the quantity in the denominator;
3. the exponent on e;
4. the quantity inside a trigonometric function.

Remember that some integrands may need to be rearranged to fit one of these cases.

(5) Section 7.3 - Area and the Definite Integral

  (i) understand the difference between indefinite and definite integrals
  (ii) approximate the area under the graph of a function using sums
  (iii) find the area under the graph of a function by integration

(6) Section 7.4 - The Fundamental Theorem of Calculus

  (i) evaluate the definite integral of a function
  (ii) find the area covered by the graph of a function over an interval

Week of 11/29/2021

(1) Section 7.5 - The Area Between Two Curves

  (i) find the area between two curves
  (ii) find points where two curves intersect and observe which curve is greater

(2) Section 8.3 - Volume and Average Value

  (i) solid of revolution
  (ii) find the volume of a solid formed by rotation about the x-axis
  $V = \int_a^b \pi[f(x)]^2 dx$
  (iii) find the average value of a function
  $\frac{1}{b-a} \int_a^b f(x)dx$

(3) Section 9.1 - Functions of Several Variables

  (i) evaluate functions with several variables
  (ii) graph functions with several variables
  (iii) traces
  (iv) level curves (side note: more general, these are called level sets for a function of n variables that takes on some constant. level sets with 2 independent variables are called level curves. level sets with 3 independent variables are called level surfaces.)

Practice problems from section 6.3

(1) Find dy/dx by implicit differentiation for the following.

(a) $6x^2 + 5y^2 = 36$
(b) $8x^2 - 10xy + 3y^2 = 26$
(c) $2\sqrt{x} + 4\sqrt{y} = 5y$
(d) $e^{x^2y} = 5x + 4y + 2$
(e) $x + \ln y = x^2y^3$
(f) $\sin(xy) = x$

(2) Find the equation of the tangent line at the given point on each curve.

(a) $x^2 + y^2 = 25; \quad (-3, 4)$
(b) $x^2y^2 = 1; \quad (-1, 1)$
(c) $y + \frac{\sqrt{x}}{y} = 3; \quad (4, 2)$

Practice problems from section 6.5

(1) Find dy for the given values of x and Δx.

(a) $y = 2x^3 - 5x; \quad x = -2,\ \Delta x = 0.1$
(b) $y = x^3 - 2x^2 + 3; \quad x = 1,\ \Delta x = -0.1$
(c) $y = \sqrt{3x + 2}; \quad x = 4,\ \Delta x = 0.15$
(d) $y = \frac{2x - 5}{x + 1}; \quad x = 2,\ \Delta x = -0.03$

(2) Use the differential to approximate each quantity. Then use a calculator to approximate the quantity, and give the absolute value of the difference in the two results to 4 decimal places.

(a) $\sqrt{145}$
(b) $\sqrt{0.99}$
(c) $e^{0.01}$
(d) $\ln 1.05$
(e) $\sin(0.03)$

(3) The concentration of a certain drug in the bloodstream t hours after being administered is approximately

$$C(t) = \frac{5t}{9 + t^2}.$$

Use the differential to approximate the changes in concentration for the following changes in t.

(a) 1 to 1.5
(b) 2 to 2.25

(4) The radius of a blood vessel is 1.7 mm. A drug causes the radius to change to 1.6 mm. Find the approximate change in the area of a cross section of the vessel. (*Hint:* Area of a cross section of the blood vessel is πr^2.)

(5) The shape of a colony of bacteria on a Petri dish is circular. Find the approximate increase in its area if the radius increases from 20 mm to 22 mm.

Practice problems from section 7.1

(1) What must be true of $F(x)$ and $G(x)$ if both are antiderivatives of $f(x)$?

(2) Explain why the restriction $n \neq -1$ is necessary in the rule $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$.

(3) Find the following.

(a) $\int (5x(x^2 - 8))dx$

(b) $\int \dfrac{1}{3x^2} dx$

(c) $\int 3e^{-0.2x} dx$

(d) $\int (x+1)^2 dx$

(4) Find an equation of the curve whose tangent line has a slope of $f'(x) = x^{2/3}$ given that the point $(1, 3/5)$ is on the curve.

(5) According to Fick's law, the diffusion of a solute across a cell membrane is given by

$$c'(t) = \frac{kA}{V}\left[C - c(t)\right], \tag{1}$$

where A is the area of the cell membrane, V is the volume of the cell, $c(t)$ is the concentration inside the cell at time t, C is the concentration outside the cell, and k is a constant. If c_0 represents the concentration of the solute inside the cell when $t = 0$, then it can be shown that

$$c(t) = (c_0 - C)e^{-kAt/V} + C. \tag{2}$$

(a) Use the last result to find $c'(t)$.

(b) Substitute back into Equation (1) to show that (2) is indeed the correct antiderivative of (1).

(6) Under certain conditions, the number of cancer cells $N(t)$ at time t increase at a rate

$$N'(t) = Ae^{kt}$$

where A is the rate of increase at time 0 (in cells per day) and k is a constant.

(a) Suppose $A = 50$, and at 5 days, the cells are growing at a rate of 250 per day. Find a formula for the number of cells after t days, given that 300 cells are present at $t = 0$.

(b) Use your answer from part (a) to find the number of cells present after 12 days.

Practice problems from section 7.2

(1) Integration is related to what differentiation method? What type of integrand suggests using integration by substitution?

(2) Find each integral using substitution.

(a) $\int (3x^2 - 5)^4 2x dx$

(b) $\int \frac{x^2}{2x^3+1} dx$

(c) $\int \frac{2x+2}{(x^2+2x-4)^4} dx$

(d) $\int z\sqrt{4z^2 - 5} dz$

(e) $\int 3x^2 e^{2x^3} dx$

(f) $\int (1-t)e^{2t-t^2} dt$

(g) $\int \frac{e^{1/z}}{z^2} dz$

(h) $\int p(p+1)^5 dp$

(i) $\int(\sqrt{x^2+12x})(x+6)dx$

(j) $\int \frac{(1+3\ln x)^2}{x}dx$

(k) $\int \frac{e^{2x}}{e^{2x}+5}dx$

(l) $\int \frac{\log x}{x}dx$

(m) $\int \cos(3x)dx$

(n) $\int \tan(6x)dx$

(o) $\int x\sin(x^2)dx$

(p) $\int \sin^7(x)\cos(x)dx$

(q) $\int 3(\sqrt{\cos(x)})(\sin(x))dx$

(r) $\int \frac{\sin(x)}{1+\cos(x)}dx$

(3) A population grows at a rate $P'(t) = 500te^{-t^2/5}$, where $P(t)$ is the population after t months.

(a) Find a formula for the population size after t months, given that the population is 2000 at $t = 0$.

(b) Use your answer from part (a) to find the size of the population after 3 months.

(4) The cost of a widget varies according to the formula $C'(t) = -\sin(t)$. At time $t = 0$, the cost is \$1. For arbitrary time t, determine a formula for the cost.

Practice problems from section 7.3

(1) Explain the difference between an indefinite integral and a definite integral.

(2) Let $f(x) = 2x + 5$, $x_1 = 0$, $x_2 = 2$, $x_3 = 4$, $x_4 = 6$, and $\Delta x = 2$.

(a) Find $\Sigma_{i=1}^4 f(x_i)\Delta x$.

(b) The sum in part (a) approximates a definite integral using rectangles. The height of each rectangle is given by the value of the function at the left endpoint. Write the definite integral that the sum approximates.

(3) For the below, approximate the area under the graph of $f(x)$ and above the x-axis using the following methods with $n = 4$. (i) Use left endpoints. (ii) Use right endpoints. (iii) Average the answers in parts (i) and (ii). (iv) Use midpoints.

(a) $f(x) = 2x + 5$ from $x = 2$ to $x = 4$

(b) $f(x) = -x^2 + 4$ from $x = -2$ to $x = 2$

(c) $f(x) = e^x + 1$ from $x = -2$ to $x = 2$

(d) $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 3$

(e) $f(x) = \sin(x)$ from $x = 0$ to $x = \pi$

(f) $f(x) = \ln(x)$ from $x = 1$ to $x = 5$

Practice problems from section 7.4

(1) Evaluate each definite integral.

(a) $\int_{-2}^{4}(-3)dp$

(b) $\int_{-1}^{2}(5t-3)dt$

(c) $\int_0^2(5x^2-4x+2)dx$

(d) $\int_0^4 2(t^{1/2}-t)dt$

(e) $\int_1^4(5y\sqrt{y}+3\sqrt{y})dy$

(f) $\int_4^6 \frac{2}{(2x-7)^2}dx$

(g) $\int_1^5 (6n^{-2} - n^{-3})dn$

(h) $\int_{-3}^{-2} (2e^{-0.1y} + \frac{3}{y})dy$

(i) $\int_1^2 (e^{4u} - \frac{1}{(u+1)^2})du$

(j) $\int_1^2 \frac{\ln(x)}{x}dx$

(k) $\int_0^{\pi/6} \tan(x)dx$

Practice problems from section 7.5

(1) Find the area between the curves.

(a) $x = -2$, $x = 1$, $y = 2x^2 + 5$, $y = 0$

(b) $x = -3$, $x = 1$, $y = x^3 + 1$, $y = 0$

(c) $x = -2$, $x = 1$, $y = 2x$, $y = x^2 - 3$

(d) $y = x^2 - 30$, $y = 10 - 3x$

(e) $y = x^2$, $y = 2x$

(f) $x = 1$, $x = 6$, $y = \frac{1}{x}$, $y = \frac{1}{2}$

(g) $x = -1$, $x = 1$, $y = e^x$, $y = 3 - e^x$

(h) $y = x^{4/3}$, $y = 2x^{1/3}$

(i) $y = \sqrt{x}$, $y = x\sqrt{x}$

(j) $x = 0$, $x = \frac{\pi}{4}$, $y = \cos(x)$, $y = \sin(x)$

Practice problems from section 8.3

(1) Find the volume of the solid of the revolution formed by rotating about the x-axis each region bounded by the given curves.

(a) $f(x) = x$, $y = 0$, $x = 0$, $x = 3$

(b) $f(x) = 2x + 1$, $y = 0$, $x = 0$, $x = 4$

(c) $f(x) = \frac{1}{3}x + 2$, $y = 0$, $x = 1$, $x = 3$

(d) $f(x) = \sqrt{x}$, $y = 0$, $x = 1$, $x = 4$

(e) $f(x) = e^2$, $y = 0$, $x = 0$, $x = 2$

(f) $f(x) = x^2$, $y = 0$, $x = 1$, $x = 5$

(g) $f(x) = 1 - x^2$, $y = 0$

(h) $f(x) = \sqrt{r^2 - x^2}$, $y = 0$

(2) [*This type of question will not be on the final exam.*] Find the average value of each function on the given interval.

(a) $f(x) = x^2 - 4$; $[0, 5]$

(b) $f(x) = \sqrt{x+1}$; $[3, 8]$

(c) $f(x) = e^{x/7}$; $[0, 7]$

(d) $f(x) = x^2e^{2x}$; $[0, 2]$

(e) $f(x) = \sin(x)$; $[0, \pi]$

Practice problems from section 9.1

(1) Let $f(x,y) = 2x - 3y + 5$. Find the following.

(a) $f(2,-1)$
(b) $f(-2,-3)$
(c) $f(-4,1)$
(d) $f(0,8)$

(2) Let $f(x,y) = xe^{x+y}$. Find the following.

(a) $f(1,0)$
(b) $f(2,-2)$
(c) $f(3,2)$
(d) $f(-1,4)$

(3) Let $f(x,y) = x\sin(x^2y)$. Find the following.

(a) $f(1,\frac{\pi}{2})$
(b) $f(\frac{1}{2},\pi)$
(c) $f(\sqrt{\pi},\frac{1}{2})$
(d) $f(-1,-\frac{\pi}{2})$

(4) Graph the first-octant portion of each plane.

(a) $x+y+z=9$
(b) $2x+3y+4z=12$
(c) $x+y=4$
(d) $x=5$
(e) $z=4$

(5) Graph the level curves in the first quadrant of the xy-plane for the following functions at heights of $z=0$, $z=2$, and $z=4$.

(a) $3x+2y+z=24$
(b) $3x+y+2z=8$
(c) $y^2-x=-z$
(d) $2y-\frac{x^2}{3}=z$

(6) Discuss how a function of three variables in the form $w=f(x,y,z)$ might be graphed.

(7) A graph that was not shown in this section is the hyperboloid of one sheet, described by the equation $x^2+y^2-z^2=1$. Describe it as completely as you can.

(8) Let $f(x,y) = 4x^2 - 2y^2$, and find the following.

(a) $\dfrac{f(x+h,y)-f(x,y)}{h}$
(b) $\dfrac{f(x,y+h)-f(x,y)}{h}$
(c) $\lim_{h\to 0}\dfrac{f(x+h,y)-f(x,y)}{h}$
(d) $\lim_{h\to 0}\dfrac{f(x,y+h)-f(x,y)}{h}$

(9) Let $f(x, y) = 5x^3 + 3y^2$, and find the following.

(a) $\dfrac{f(x+h, y) - f(x, y)}{h}$

(b) $\dfrac{f(x, y+h) - f(x, y)}{h}$

(c) $\lim_{h\to 0} \dfrac{f(x+h, y) - f(x, y)}{h}$

(d) $\lim_{h\to 0} \dfrac{f(x, y+h) - f(x, y)}{h}$